Climatic Structural Stability of the Kuroshio Extension Jet and Catastrophe Theory

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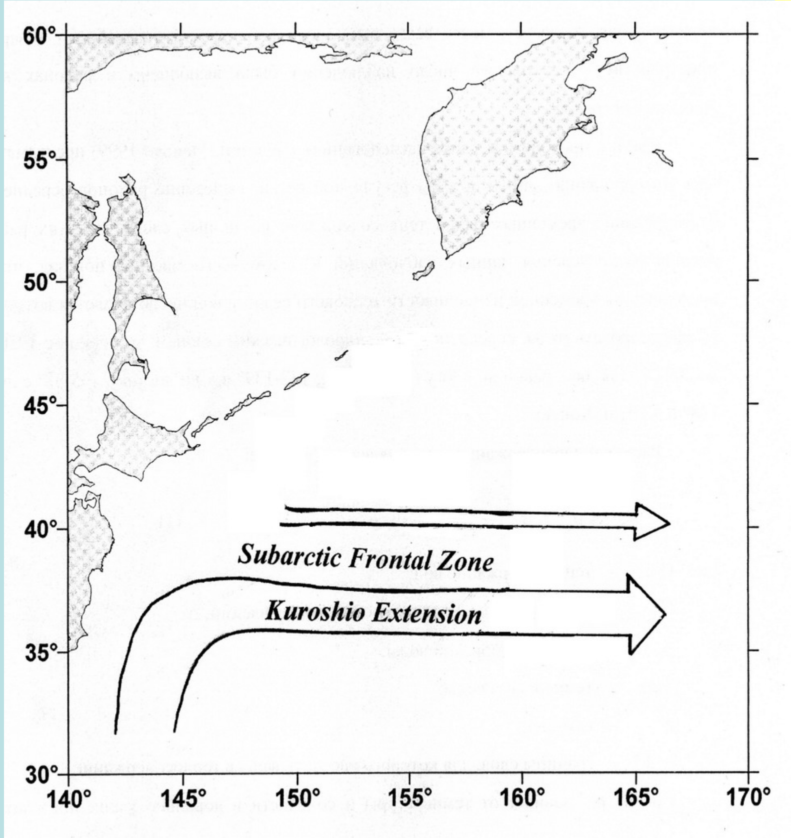
The purpose of our presentation is to show that the slow accumulation physical parameters owing climatic trend can cause a fast structural change of ocean circulation.

We show the engineering energetic method

to estimate structural instability of an ocean jet current

We use application of two physical principles:
1. The Second Law of Thermodynamics
2. The Static Structure Instability (Catastrophe Theory)

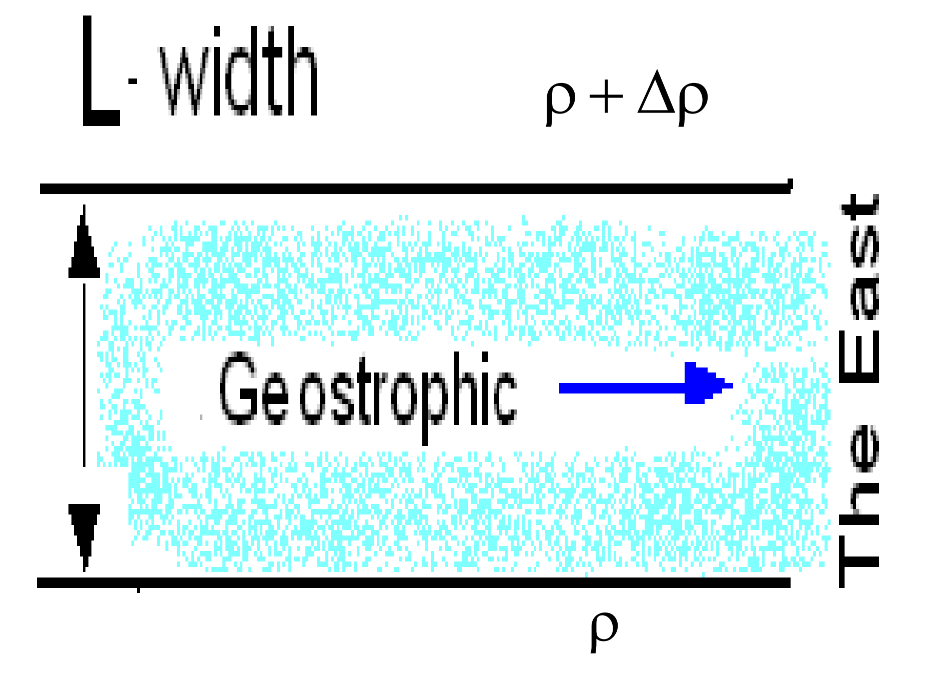
The ocean object of modelling is Jet geostrophic current applied to the Kuroshio Extension



- The Sharp zonal border divides the North Pacific on the subtropical and subarctic zones. **The cross-section of this border has two climatic fronts - Subarctic Front and density front which it is the jet current Kuroshio Extension.**

- We used the following simplifications - the climatic scale, stationary condition. The detailed structure (bottom topography, meandering, synoptic variability, etc..) were ignored.

The purpose of modelling:



- To determine conditions of stationary existence of the structure border :
- L – width of the jet current (controlled parameter)
- $\Delta\rho$ - cross difference of density (controlling parameter – in our case it is delta SST) :
- L=L ($\Delta\rho$)
- To find limiting climatic changes of parameters at which the jet current collapses as a compact structure

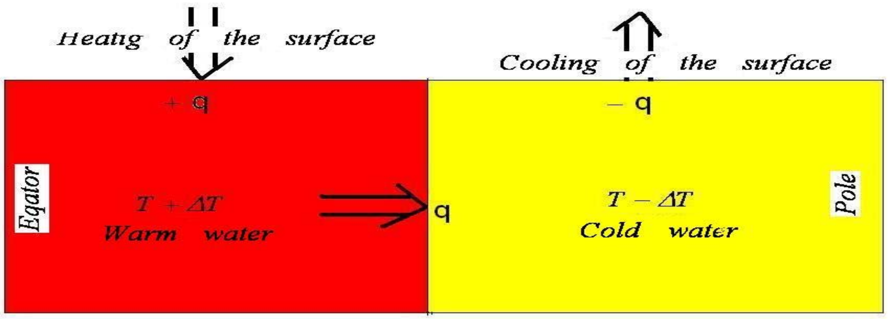
The Second Law of Thermodynamics and climatic trend of SST

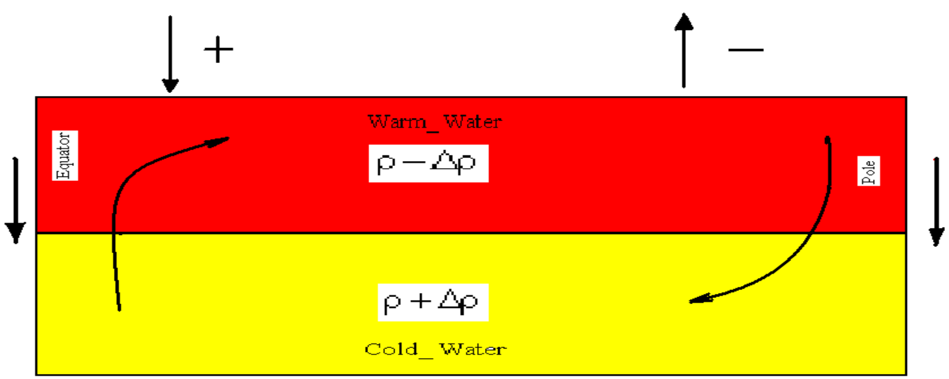
Controlling parameter – spatial distribution of density difference $\Delta\rho$ *in the direction South - North*

The main idea:

The kinetic energy production is more depend on the climatic trends of the spatial distribution of non-uniformity thermodynamic parameters (SST), **than from climatic trends of common heating - cooling of the ocean waters.**

Application of 2nd Law of Thermodynamics



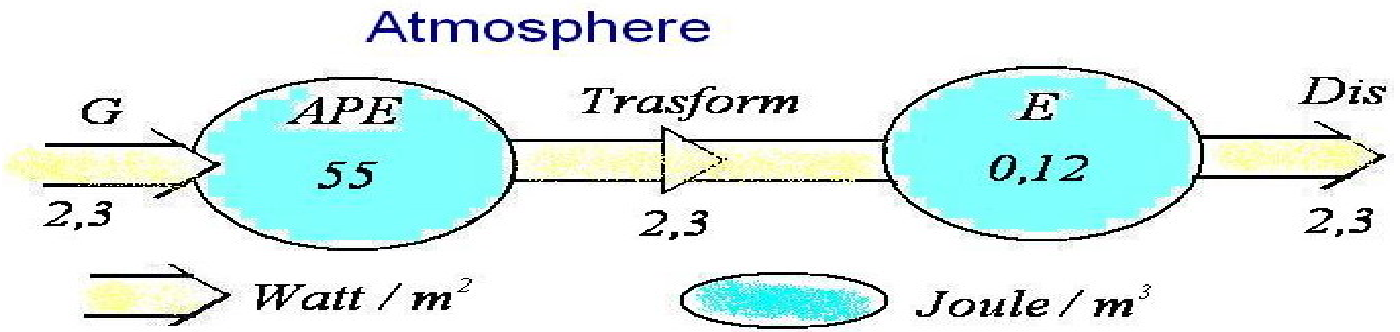


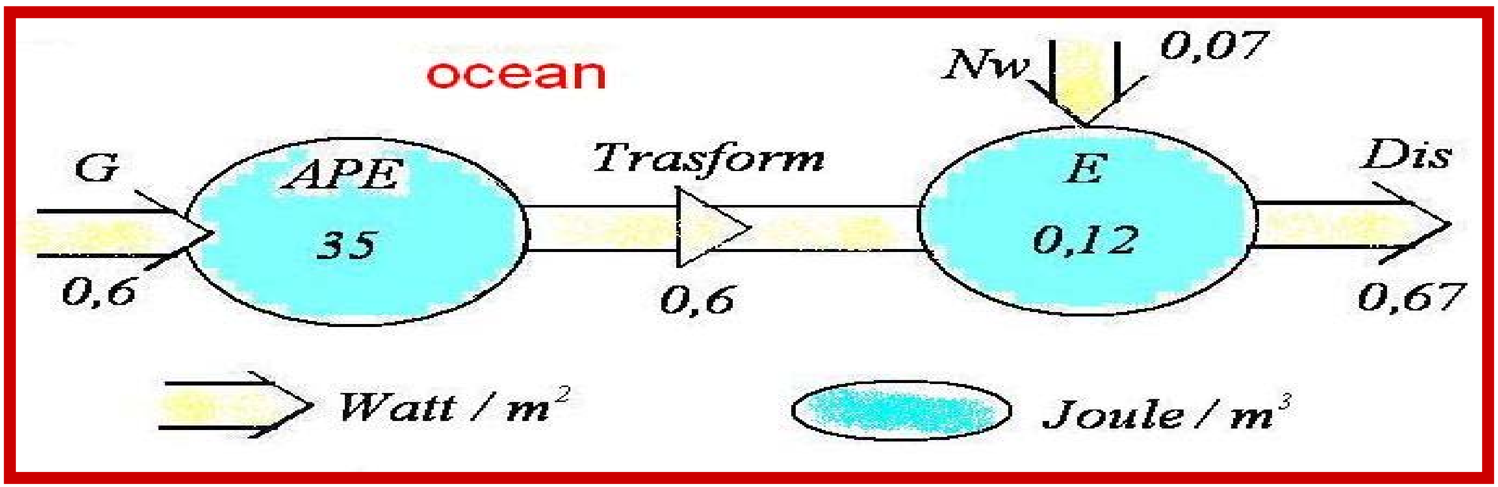
$$J = \frac{q}{T+\Delta T} + \frac{-q}{T-\Delta T} \approx -\frac{2q\Delta T}{T^2}$$

- ***Entropy Flux*** *shows the deviation of a stationary condition from an equilibrium condition*

$$Avaiblale\_Potentional\_Enerdgy = g\Delta\rho\frac{h}{8} \approx -g\alpha\Delta T\frac{h}{8}$$

Comparison of energy cycles of the atmosphere and the ocean





- **The Energy cycle in the atmosphere (Lorenz). The Energy cycle in the ocean is calculated with the help of the entropy flux. Generation (G), Transformation (Trasf), Dissipation (Diss) of energy. Available potential energy (APE), Kinetic energy (E). Wind work (Nw).**

DATA SST

1. Rayner, N.A., Brohan P., Parker D.E., Folland C.K., Kennedy J.J., Vanicek M., Ansell T., Tett S.F.B. Improved analyses of changes and uncertainties in sea surface temperature measured in situ since the mid-nineteenth century: the HadSST2 data set. - Journal of Climate. 2006. 19(3), p. 446-469.
2. http://hadobs.metoffice.com/hadsst2/

1948 1953 … … … 2003 2007

→ ***time***

1948	1953	…	2003–2007
$\overline{T}_1$	$\overline{T}_2$	…	$\overline{T}_{12}$
$\delta\overline{T}_1$	$\delta\overline{T}_2$	…	$\delta\overline{T}_{12}$

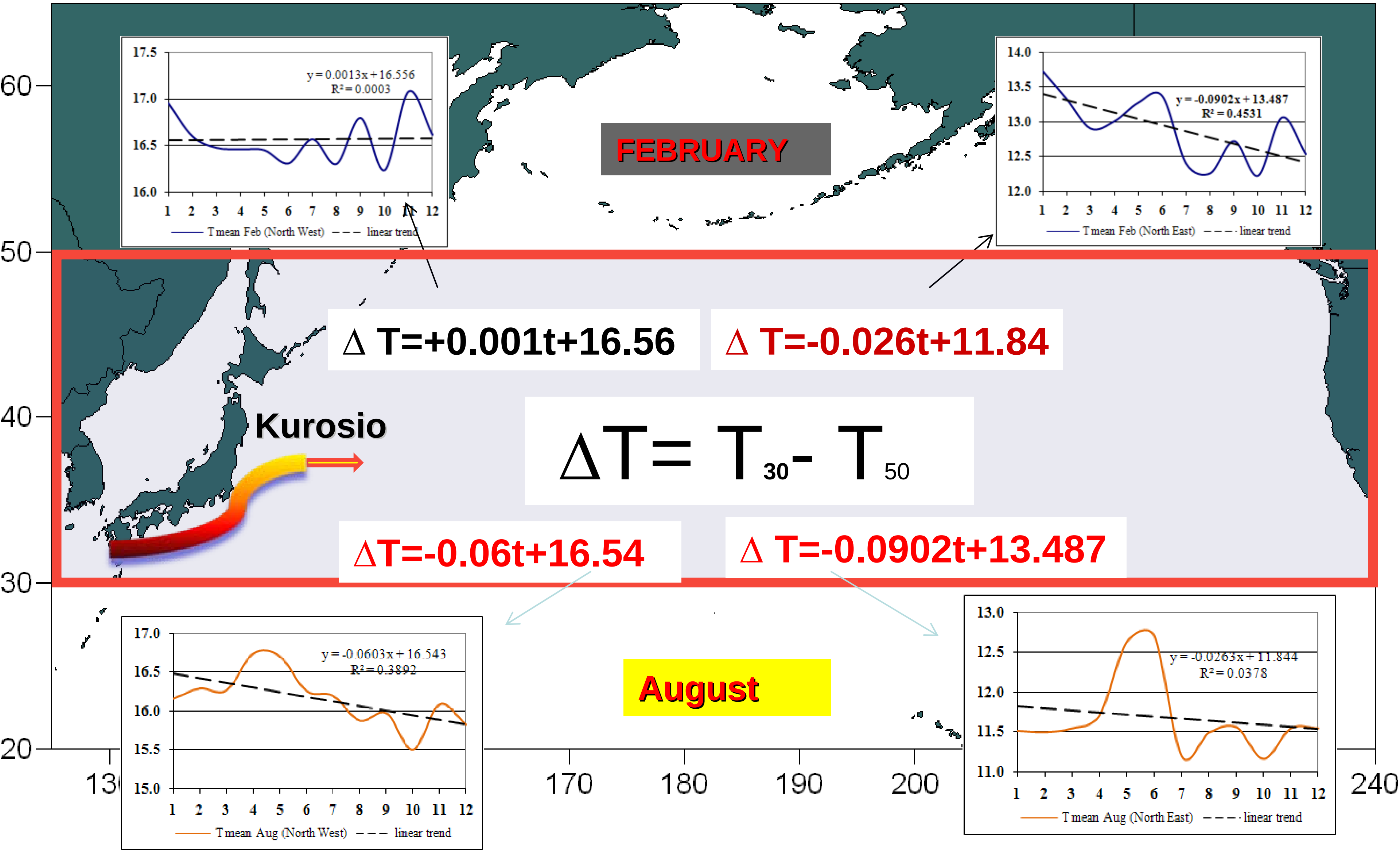
$$\overline{T}_i = \left(\frac{1}{n}\Sigma T_j\right)_i$$

\- **SST trend**

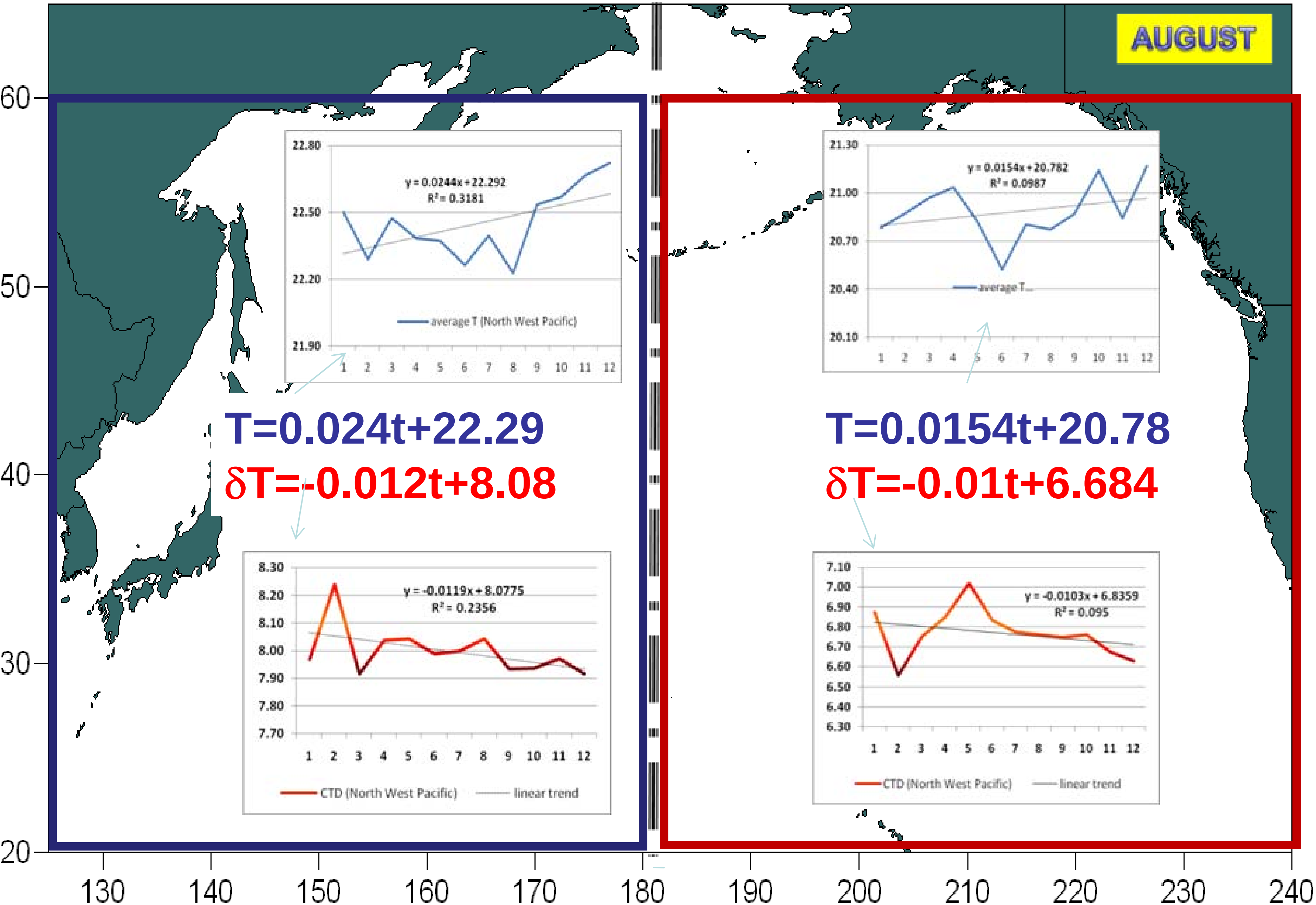
$$\delta\overline{T}_i = \sigma = \sqrt{\frac{1}{n}\sum_i (T_j - \overline{T})^2}_{\,i}$$

\- **trend of spatial heterogeneity of SST**

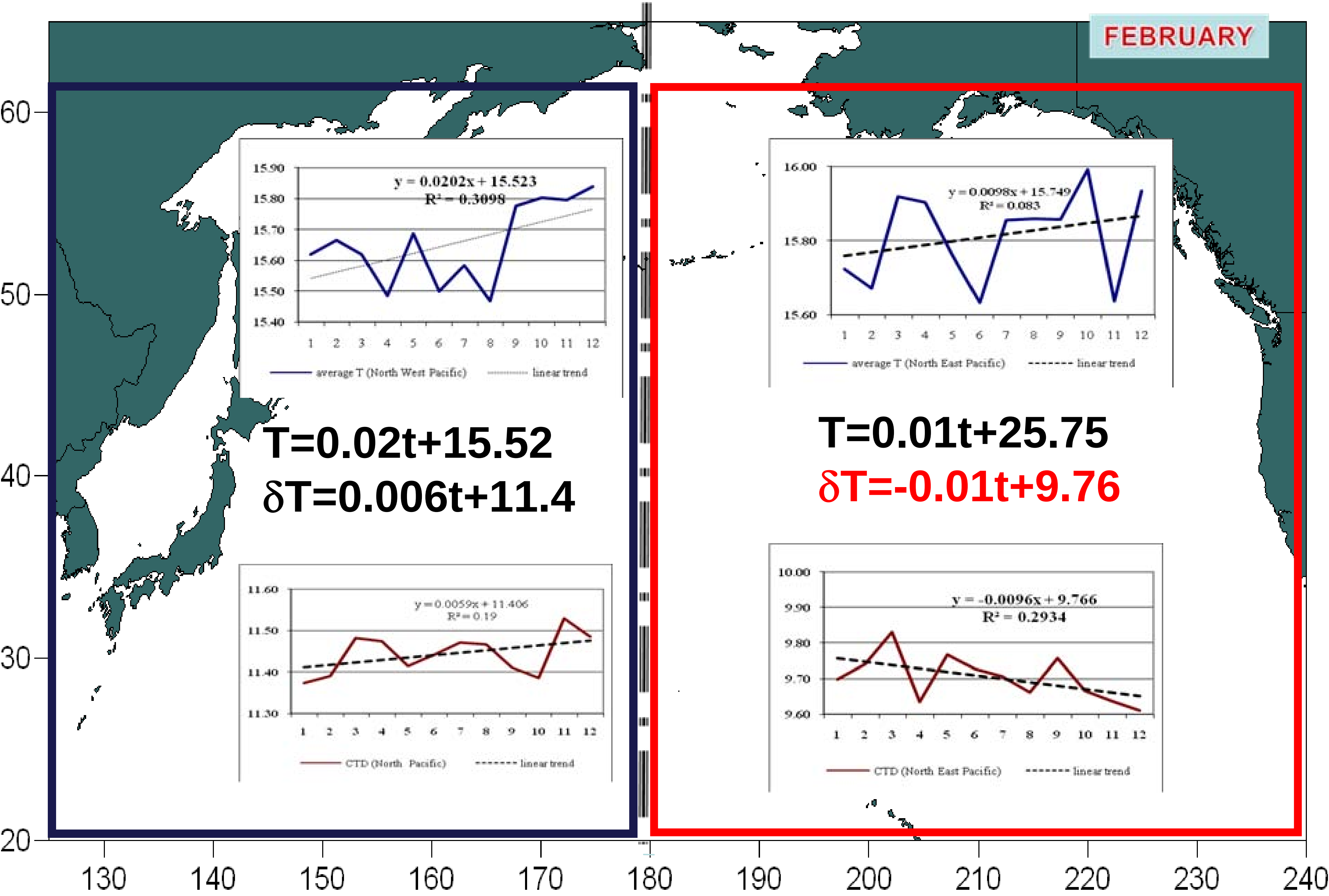
The climatic trends of the temperature difference between 30⁰N and 50⁰N (The last 60 years.   1 step= 5 years)



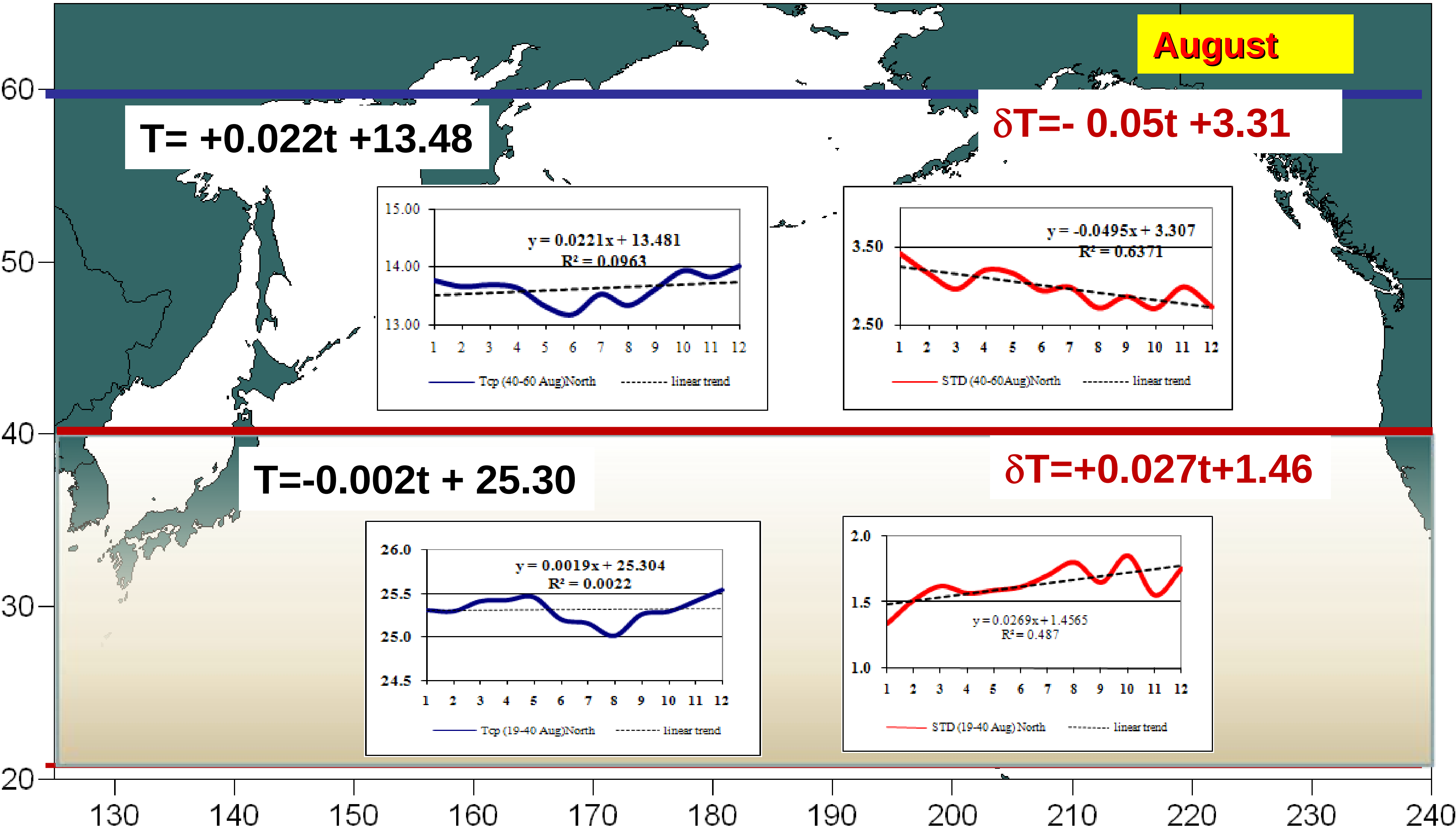
The climatic trend for the last 60 years. The Northwest Pacific. The Northeast Pacific. The Summer - reduction of delta T



The climatic trend for the last 60 years. The Northwest Pacific. The Northeast Pacific. The Winter



The Trends of T(t), ΔT(t) .
The Subarctic. The Subtropics. The Summer.



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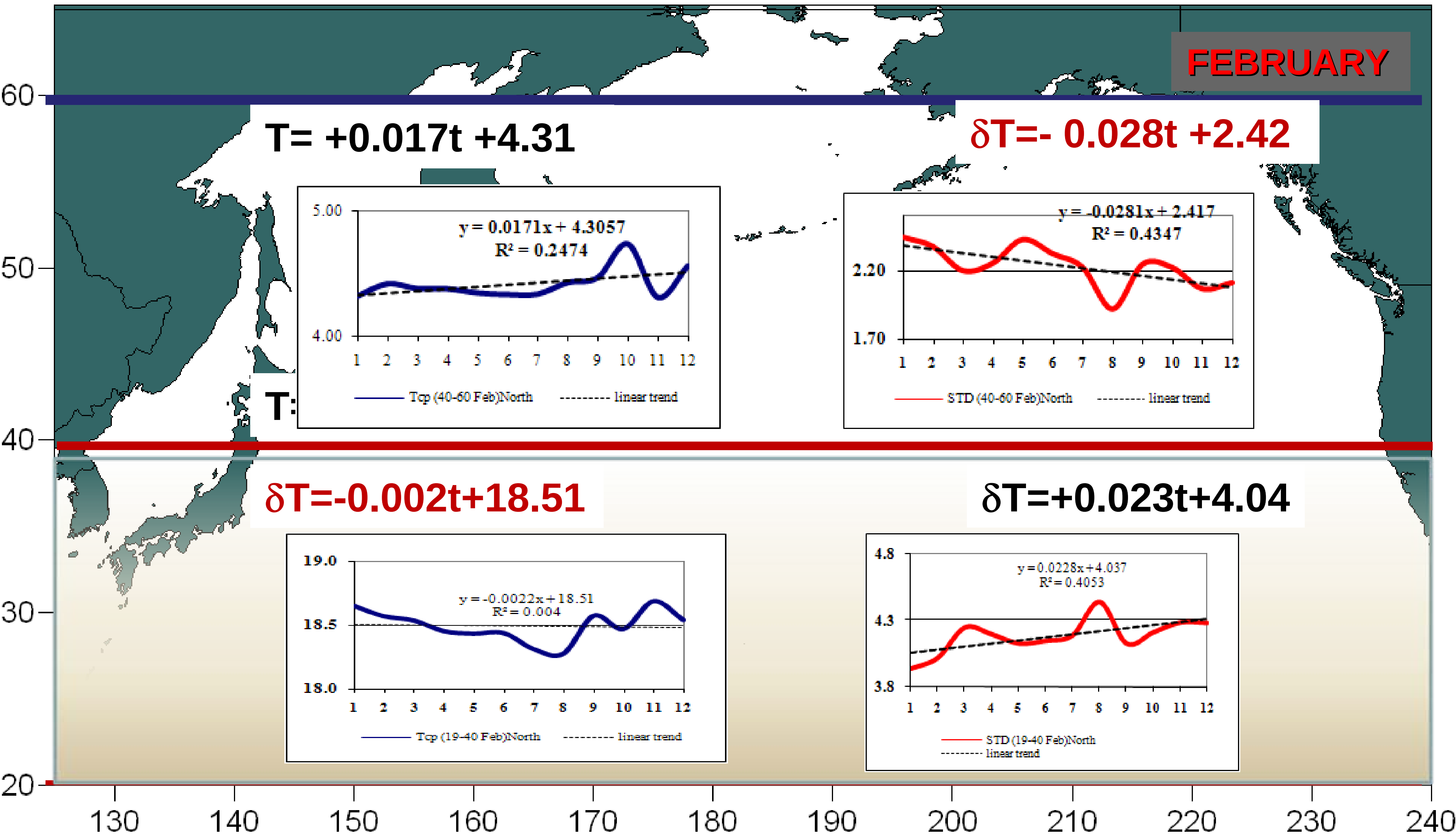
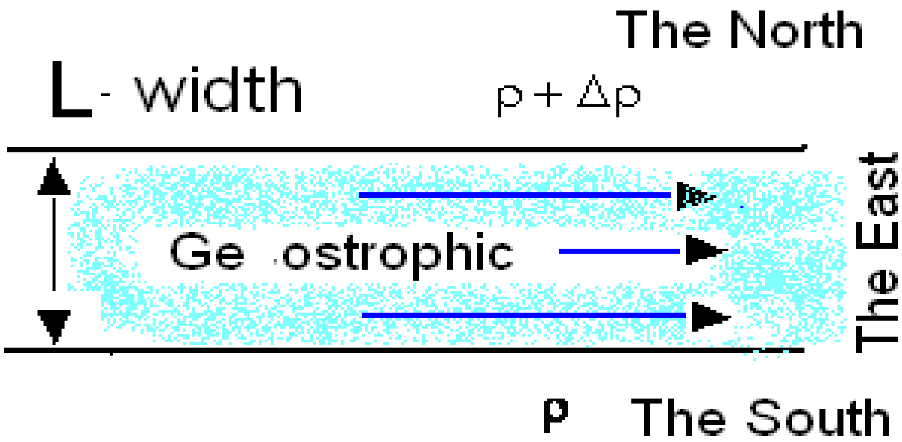


The simple energetic model (conservative system)



- Kinetic Energy….

$$KE = \frac{\rho}{2} hLv^2 = \frac{\rho}{2} hL \left(\frac{\Delta \rho g h}{\rho f L}\right)^2$$

- Potential Energy

$$PE = \Delta \rho g \frac{h^2 L}{12}$$

- Mechanical Energy

$$E(L) = KE + PE$$

Stability of structure = min E(L)

(conservative system)

From **min E(L)** follow $\frac{dE}{dL} = 0$ $\frac{d^2E}{dL^2} > 0$

and the width of jet current is

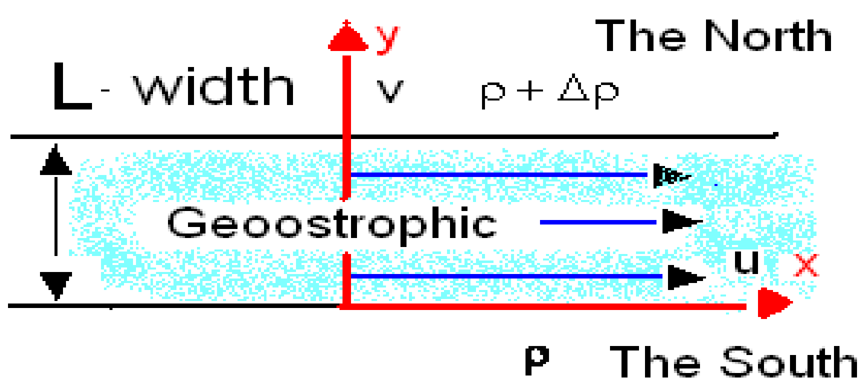
$$L = \sqrt{6}\sqrt{\frac{\Delta\rho g h}{\rho f^2}} \approx const * R$$

R – the Rossby deformation radius (classic Stommel's model of Gulf Stream - principle of conservation of potential vorticity).

This conservative system is never unstable

The Energetic Model - Non-conservative system. Balance of forces across current

- (+)Inertial (convergence) force vU_y
- (-)Turbulent mixing force $(AU_y)_y$
- (:)Coriolis force $-fv$
- (+)Geostrophic balance $U = \dfrac{\Delta\rho g h}{\rho f L} = R^2 \dfrac{f}{L}$



The equivalent variational formulation of motion's equations

- *The local potential of. P. Glandsdorff and I. Prigogine*

$$P(u,u_c,v,v_c,w,w_c,p,p_c)=$$

$$\int_{-H}^{0} dz \int_{y_1}^{y_2} [\frac{A}{2} u_y^2 + v_c u_{cy} u - f(v_c u + u_c v) + \frac{p_{cy}}{\rho_0} v + w(\frac{p_{cz}}{\rho} + g) + \frac{1}{\rho_0}(u_c p_x + v_c p_y)]dy$$

- *with a stationary condition the fields of density, pressure, and speed have the such configuration at which functional accepts minimal value.* The Euler equations identically coincide with the model's equations

- $[\frac{\delta P}{\delta u}]_{u_c}=0,\quad [\frac{\delta P}{\delta v}]_{v_c}=0,\quad [\frac{\delta P}{\delta w}]_{w_c}=0,\quad [\frac{\delta P}{\delta p}]_{p_c}=0$

$$\rho_0 fu=-p_y \qquad vu_y - fv = (Au_y)_y \qquad 0=-p_z-\rho g \qquad w_z+v_y=0$$

The physical content – a stationary condition corresponds to the generalized minimal entropy production (local dissipation of energy) - I. Prigogine.

- **Operated parameter – the cross difference of density to the North and the South of the channel** (in our case it is delta SST) **.**
- **Control parameter - width of the jet current (channel)**

  L= L($\Delta\rho$ **)**

- *External parameters of model:* **A** *- horizontal turbulent coefficient across the channel,* **V** *- intensity of convergence.*

It is important - the model is not closed. (The model has 4 equations and 5 unknown parameters). Our purpose is energy estimations only.

Stationary condition of dissipative structure – **minimum P**

- **The integration and calculation of local potential in simple**

$$P_s(L,L_c)=\frac{hA}{8L^3}\left(\frac{g'h}{f}\right)^2+\frac{hv_0}{3L_c(L_c+2L)}\left(\frac{g'h}{f}\right)^2-\frac{hv_0 g'hL_c}{2(L_c+L)}-\frac{hA}{2L_c^2L}\left(\frac{g'h}{f}\right)^2$$

- **From condition of P = minimum** $\left[\frac{\partial P_s}{\partial L}\right]_{L_C}=0$ **follows the cubic equation**

$$\left(\frac{L}{R}\right)^3-\frac{16}{27}\left(\frac{L}{R}\right)+\frac{A}{v_0 R}=0$$

- **where** $R=\frac{\sqrt{g'h}}{f}$ **the Rossby deformation radius**

The roots of the cubic equation. The interpretation of physical result

- *The positive roots of cubic equation give us a jet current with positive width* **L**. *The width is*

$$L = \frac{8}{9} R \cos\frac{\varphi}{3} \qquad \cos\varphi = -\left(\frac{9}{4}\right)^3 \frac{A}{2v_0 R}$$

The particular case A=0 – the width of the jet is proportional to the Rossby scale $R = \frac{\sqrt{g'h}}{f}$ (Stommel's model for the Gulf Stream jet)

- **If the controlling parameter $\Delta\rho$ is smaller of a critical value than the positive root does not exist. The jet current is absent .**

- *There is a critical value when we lose* **a jet current**

$$-\rho\alpha\Delta T \approx \Delta\rho \leq \Delta\rho_{crit} \approx \rho A^2 f^2 (v^2 gh)^{-1}$$

Critical parameter is the bifurcation point.
This is a possible to interpret as catastrophe of the fold.

The non-conservative model has shown the following results

- There is a critical value of parameter beyond which a collapse of the jet occurs.
  Control parameter $L \leq 0.$

- In the present time, the value of controlling parameter is $\Delta\boldsymbol{\rho} > \Delta\boldsymbol{\rho}_{critl}$ and the Kuroshio Extension as the jet geostrophic current exists.

$$-\rho\alpha\Delta T_{crit} \approx \Delta\rho_{crit} \approx \rho A^2 f^2 (v^2 gh)^{-1}$$

The estimate $\Delta\rho_{crit} \approx 10^{-3}\rho$ has no practical applications

The model energetic estimations have shown the following result

- One of the possible scenarios of climatic trend is a smoothing of the density spatial gradient in the direction South-North. It causes the APE reduction, the kinetic energy (KE) reduction.
- In this case there will be a critical reduction of the meridional density gradient, then, in turn, a structural instability of ocean circulation (catastrophe). So the Kuroshio Extension could lose the property of a jet current

Thank you

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