A variational model of a jet current applied to the Kuroshio Extension

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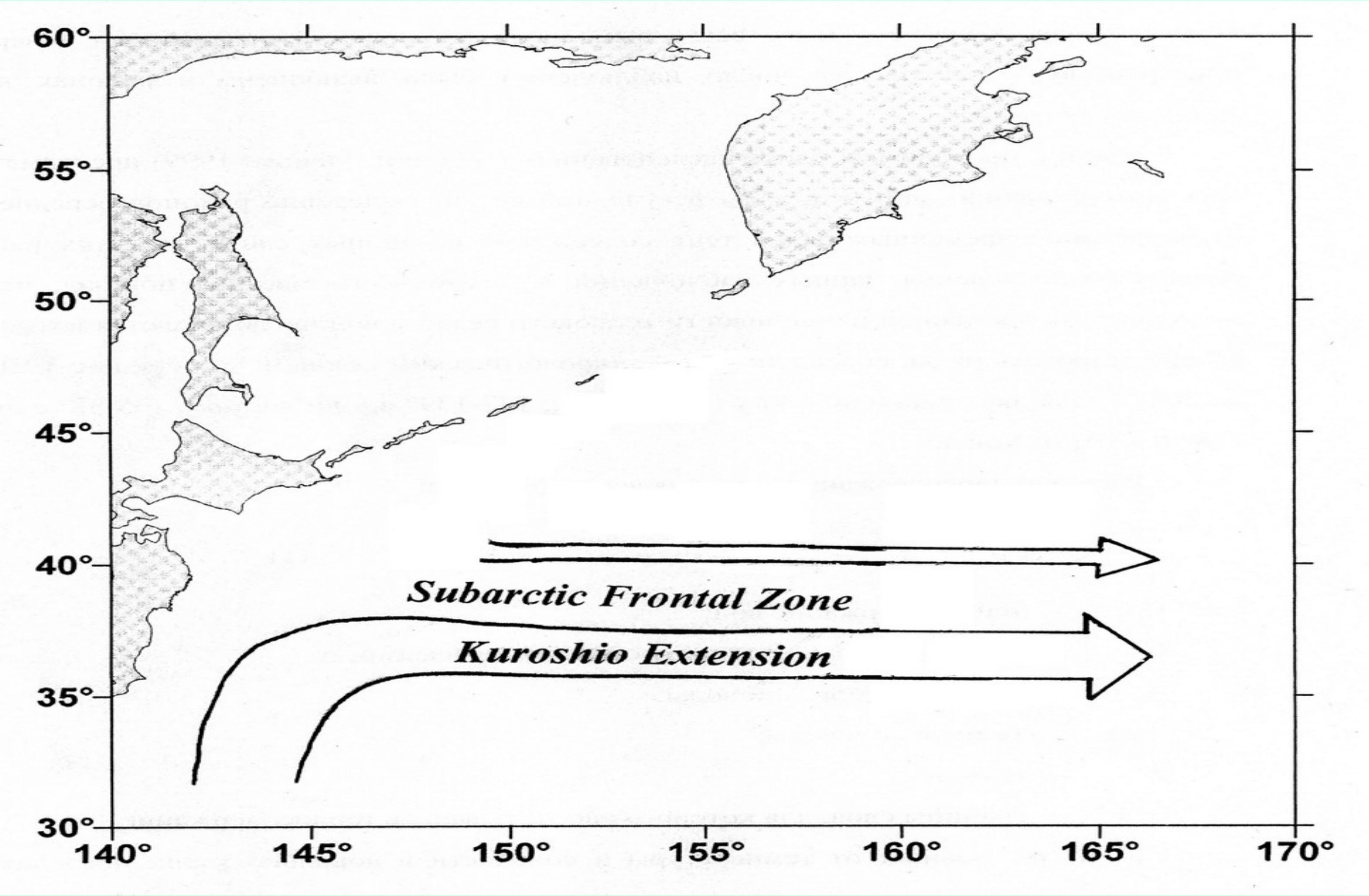
The Object of modelling – Jet geostrophic current Applied to the Kuroshio Extension (the Gulf Stream)

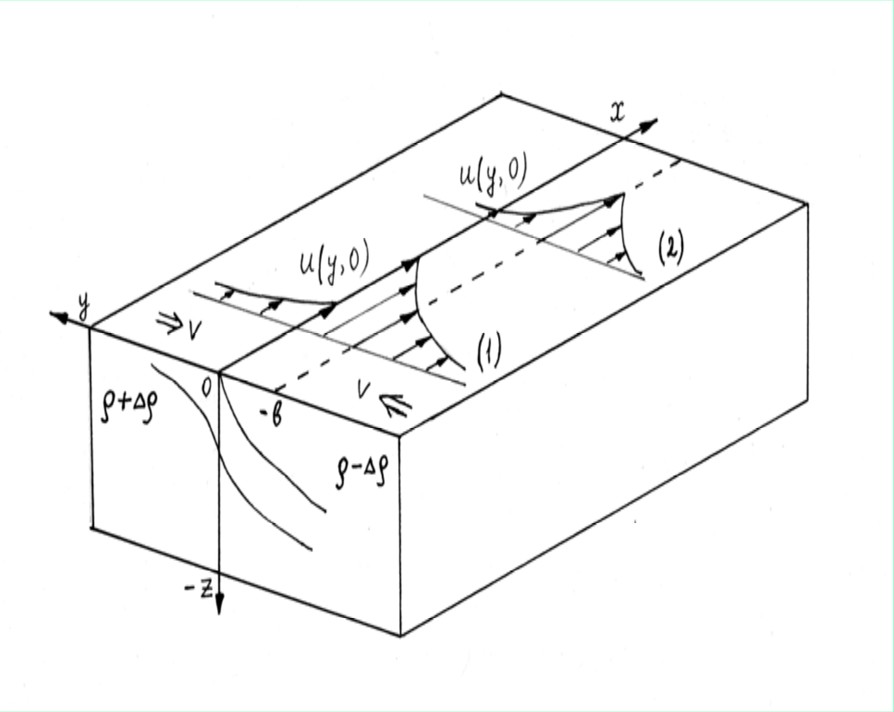


The purpose of modelling:

- To determine conditions of stationary existence of the structure border

- To find limiting climatic changes of parameters at which the jet current collapses as a compact structure

Model's geometry



- **1. A zonal channel at the f-plane**
- **2. Axis X – to the East, Axis Y – to the North, Axis Z directed upwards**
- ***Climatic parameters of the model:***
- ***∆ρ - density difference from the North to the South***
- ***A - turbulent horizontal viscosity across the channel***
- ***Vo - speed of convergence***

The equations of model

- 1. Axis Z - the hydrostatic equation

$$0 = -p_z - \rho g$$

- 2. An axis X – geostrophic approach

$$\rho_0 f u = -p_y$$

- 3. Axis Y - balance of advection forces, Coriolis forces, horizontal turbulent forces

$$v u_y - f v = (A u_y)_y$$

- 4. The equation of continuity

$$w_z + v_y = 0$$

The equivalent variational formulation of set of equations

- *The local potential of. P. Glandsdorff and I. Prigogine*

$$P(u,u_c,v,v_c,w,w_c,p,p_c)=$$

,

$$\int_{-H}^{0} dz \int_{y_1}^{y_2} [\frac{A}{2} u_y^2 + v_c u_{cy} u - f(v_c u + u_c v) + \frac{p_{cy}}{\rho_0} v + w(\frac{p_{cz}}{\rho} + g) + \frac{1}{\rho_0}(u_c p_x + v_c p_y)]dy$$

,

- *with a stationary condition the fields of density, pressure, and speed have the such configuration at which functional accepts minimal value.* The Euler equations identically coincide with the model's equations

- $$[\frac{\delta P}{\delta u}]_{u_c} = 0, \quad [\frac{\delta P}{\delta v}]_{v_c} = 0, \quad [\frac{\delta P}{\delta w}]_{w_c} = 0, \quad [\frac{\delta P}{\delta p}]_{p_c} = 0$$

$$\rho_0 f u = -p_y \qquad v u_y - f v = (A u_y)_y \qquad 0 = -p_z - \rho g \qquad w_z + v_y = 0$$

The physical content – a stationary condition corresponds to the minimal entropy production (I. Prigogine).

- **Boundary conditions – the density difference is given to the North and the South of the channel .**
- **External parameters of model: A - horizontal turbulent coefficient across the channel, Vo - intensity of convergence** V_o
- **This is important - model has 4 equations and 5 unknown parameters. The model is not closed.**
- **To close the system of equations we add a physical information on really observable fields. These fields are given in a parametrical views.**

Parametrical installation of the fields

- 1) To describe the really observable fields
- 2) Allowable physical changes of parameters should include or exclude an opportunity of existence of the structure studied (jet current)

- Here we shall show a variant of the model with two variable parameters: **L - width of the jet** current to the south from the convergence axis (the Kuroshio Extension) and Ls - width of current to the north from the convergence axis

Mathematical parameterization

- **Density field**

$$\rho(y,z,L,L_N) = \begin{cases} \rho_0 + \Delta\rho - \Delta\rho(2 - \exp(y/L))\exp(z/h), \\ \rho_0 + \Delta\rho(1 - \exp(-y/L_N))\exp(z/h), \end{cases}$$

- **Pressure field**

$$p = -g\int_0^{-z} \rho dz$$

- **Velocity field across the channel (convergence)**

$$v(y,z,L,L_N) = \begin{cases} v_0 \exp(y/L)\exp(z/h), & y \le 0, \\ -v_0 \exp(-y/L_N)\exp(z/h), & y > 0. \end{cases}$$

- **Vertical speed**

$$w(y,z,L,L_N) = -v_y(y,0,L,L_N)h\exp(z/h)$$

Mathematical parameterization

- The geostrophic component of velocity along the channel

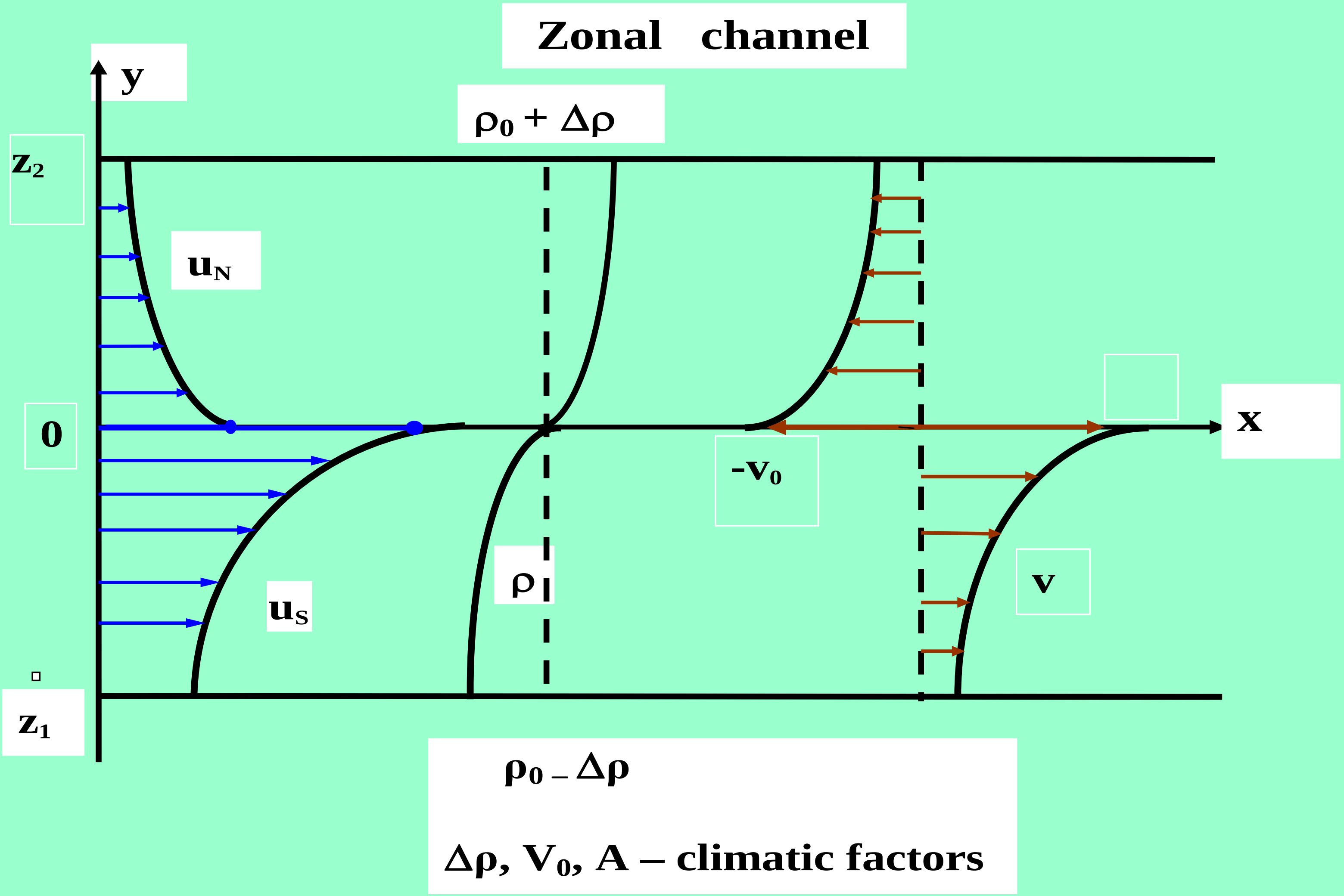
$$u(y,z,L,L_N) = \begin{cases} u_0 \exp(y/L)\exp(z/h), & y \le 0, \\ u_{0N} \exp(-y/L_N)\exp(z/h), & y > 0, \end{cases}$$

$$u_0 hL = u_{0N} hL_N = \frac{g' h^2}{2f}$$

- The integrated flux across the channel is a constant and it does not depend on the value of the parameters

$$Q = \frac{g' h^2}{f} \qquad g' = g\Delta\rho/\rho_0$$

The geometrical interpretation of the parameterization



The direct variation method for calculation of parameters

- **The integration and calculation of local potential**
- **(an example z <0) gives**

$$P_s(L,L_C) = \frac{hA}{8L^3}(\frac{g'h}{f})^2 + \frac{hv_0}{3L_C(L_C+2L)}(\frac{g'h}{f})^2 - \frac{hv_0 g'hL_C}{2(L_C+L)} - \frac{hA}{2L_C^2 L}(\frac{g'h}{f})^2$$

- **Condition of P = minimum.**
- **From the Euler equation** $[\frac{\partial P_s}{\partial L}]_{L_C} = 0$ **follows the cubic equation**

$$(\frac{L}{R})^3 - \frac{16}{27}(\frac{L}{R}) + \frac{A}{v_0 R} = 0$$

$$(\frac{L_N}{R})^3 + \frac{16}{27}(\frac{L_N}{R}) - \frac{A}{v_0 R} = 0$$

- **where** $R = \frac{\sqrt{g'h}}{f}$ **the Rossby deformation radius**

The roots of the cubic equation. The interpretation of physical result

- *There is a critical value* $\alpha = \frac{v_0 R}{A} < \frac{1}{2}\left(\frac{9}{4}\right)^3$ *when we lose the positive roots of cubic equation*
- **A big** α **gives us a jet current** (to the south of an axis of convergence). **The width is** $L = \frac{8}{9} R \cos\frac{\varphi}{3}$**,** $\cos\varphi = -\left(\frac{9}{4}\right)^3 \frac{A}{2 v_0 R}$
- **The particular case A=0 – the width of the jet is proportional to the Rossby scale.**
- (Stommel model for the Gulf Stream jet) $R = \frac{\sqrt{g'h}}{f}$
- **The opposite case. If** α **is small then the positive root** L **does not exist. The jet current (to the south of an axis of convergence) is absent**

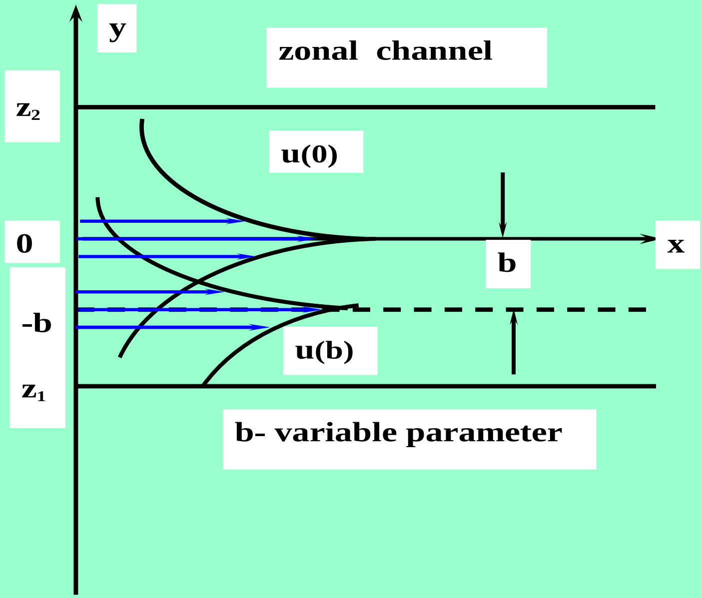
The version of the variational model with a given form of a jet current

A Position of jet axis concerning to the convergence axis

The Kuroshio Extension axis is located to the south from axis convergence

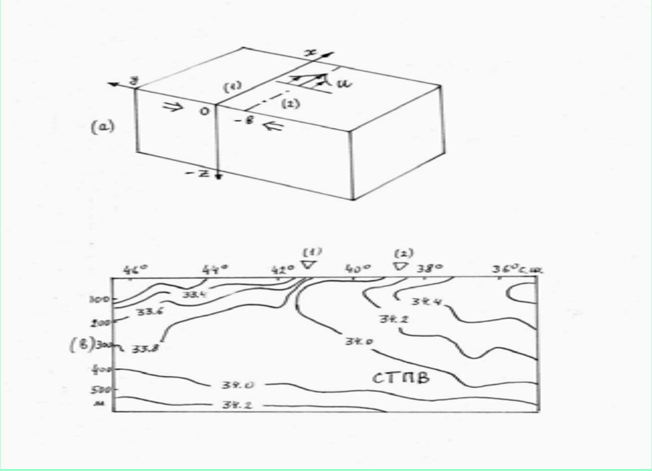
- It is the model - the width and the form of a jet is fixed, the axis of current may move in the direction to the south - the north

$$u(y,z,b) = \begin{cases} u_0 \exp((y+b)/L)\exp(z/h) \\ u_0 \exp(-(y+b)/L)\exp(z/h) \end{cases}$$



- There is only one variable

  parameter - b



Result of the variation modeling

- The local potential is written

$$P(b,b_C) = \frac{hAu_0^2}{2L} + \frac{hv_0u_0^2}{9}(\exp\frac{b_C - 2b}{L}) + 4\exp(-\frac{b+b_C}{L}) - 3\exp(-\frac{b_C}{L})) - \frac{hfv_0u_0b}{2}\exp(-\frac{b}{L})$$

The necessary condition of the minimum (Euler equation) is

$$\frac{4}{9}Ro(1+2\exp(-\frac{b}{L})) = \frac{b}{L} - 1$$

$$Ro = \frac{u_0}{fL}$$ - Rossby number

- The root of the last equation $b > 0$. This denotes the necessary of southward displacement of the jet axis from the convergence axis

The variational models have shown the following results

- There is a critical value of parameter $\alpha = \frac{v_0 \sqrt{g'h}}{Af}$ beyond which a collapse of the jet occurs

- In the present time, the value **alpha>alpha critical** and the jet geostrophic current (the Kuroshio Extension, the Gulf Stream) exists.

- The shape of the persistent jet is asymmetric with a sharp northern border.

- The position of the dynamic current axis is displaced to the south from the convergence axis. This is one of the mechanisms of formation of two-(multi) front structures in the ocean.
- It means that the geostrophic front of the Kuroshio Extension is permanently displaced to the south from to the thermohaline Subarctic Front regardless of synoptic variations.

The variational model has shown the following result

- One of the possible scenarios of climatic trend is warming, when temperature increases more considerably at high latitudes than in the tropics. In this case there will be a reduction of the meridional density gradient when, in turn, a critical value of ***alpha*** parameter. So, the Kuroshio Extension (or the Gulf Stream) can lose the property of a jet current