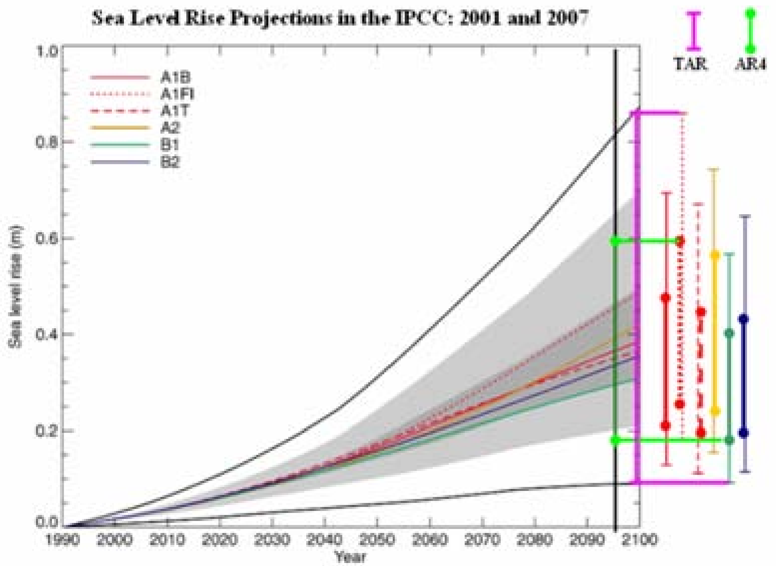
FORECASTING THE SEASONAL TO INTERANNUAL VARIABILITY OF EXTREME SEA LEVELS

Melisa Menéndez, Fernando Méndez, Iñigo J. Losada

Instituto de Hidráulica Ambiental, IHCantabria,
Universidad de Cantabria

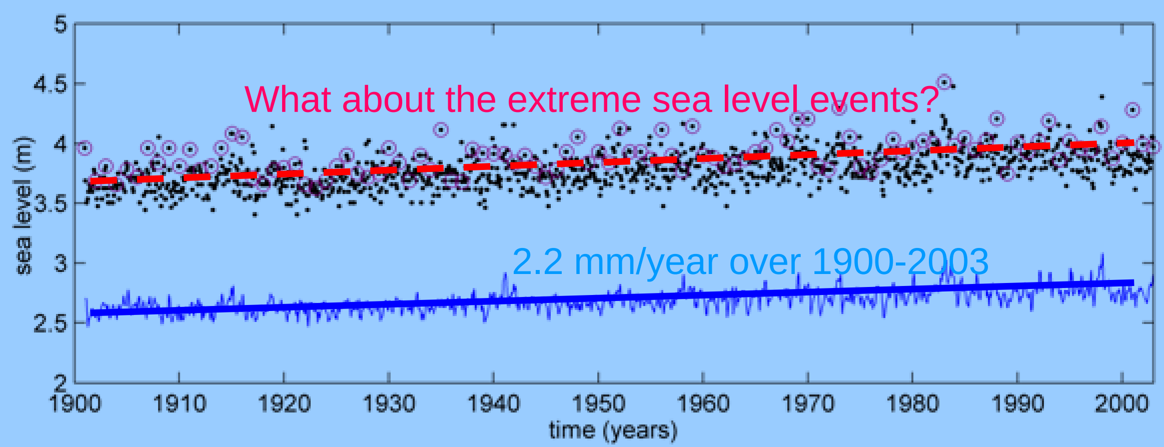
Importance of extreme sea level values..



Global annual mean sea level

1.8 mm/year over 1961-2003
3.1 mm/year over 1993-2003

(San Francisco tidal gauge time series, NOAA, station #9414290)



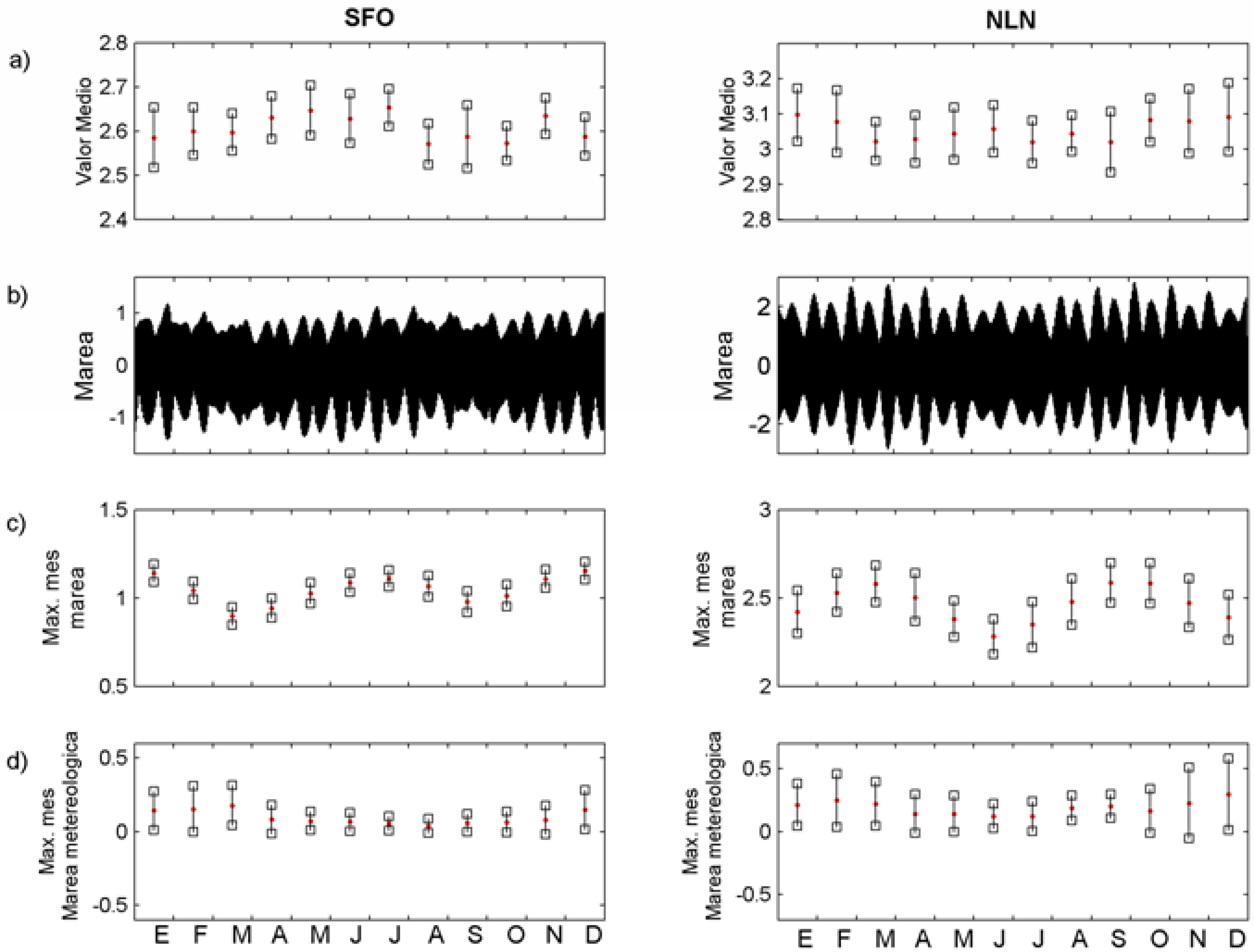
San Francisco
tidal gauge

Data set from NOAA
(California, USA)
(1901-2003)
103 years

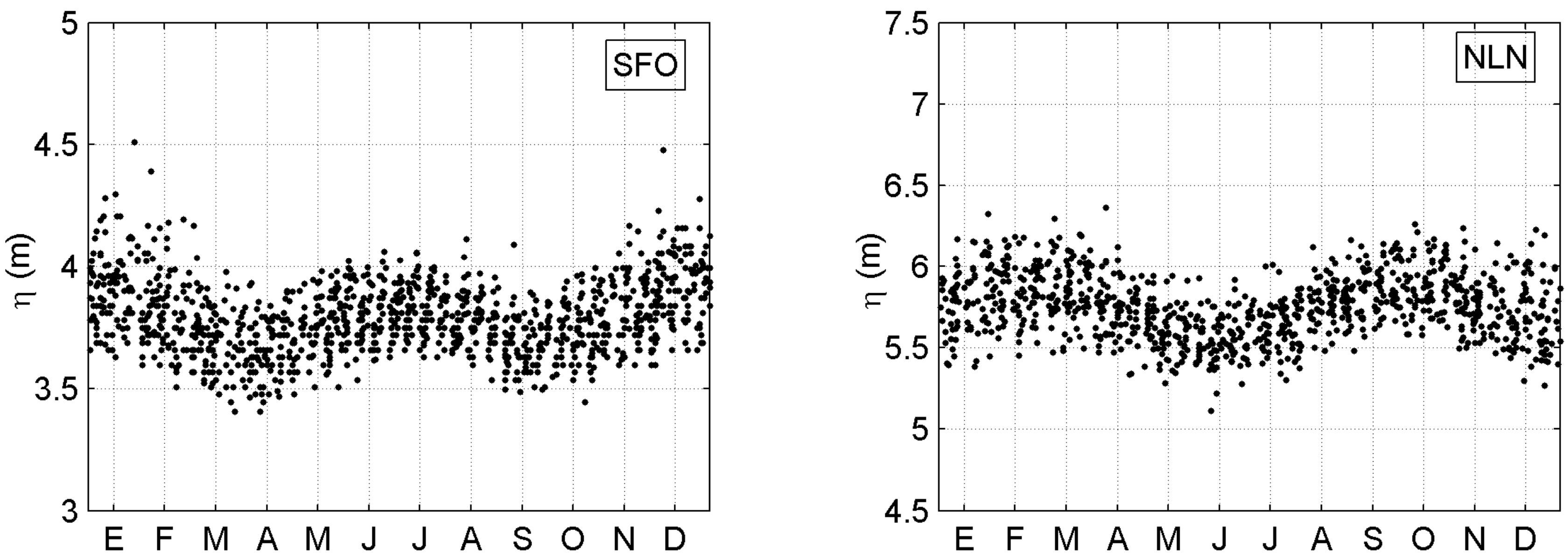
Newlyn
tidal gauge

Data set from BODC
(SW England)
(1915-2001)
87 years

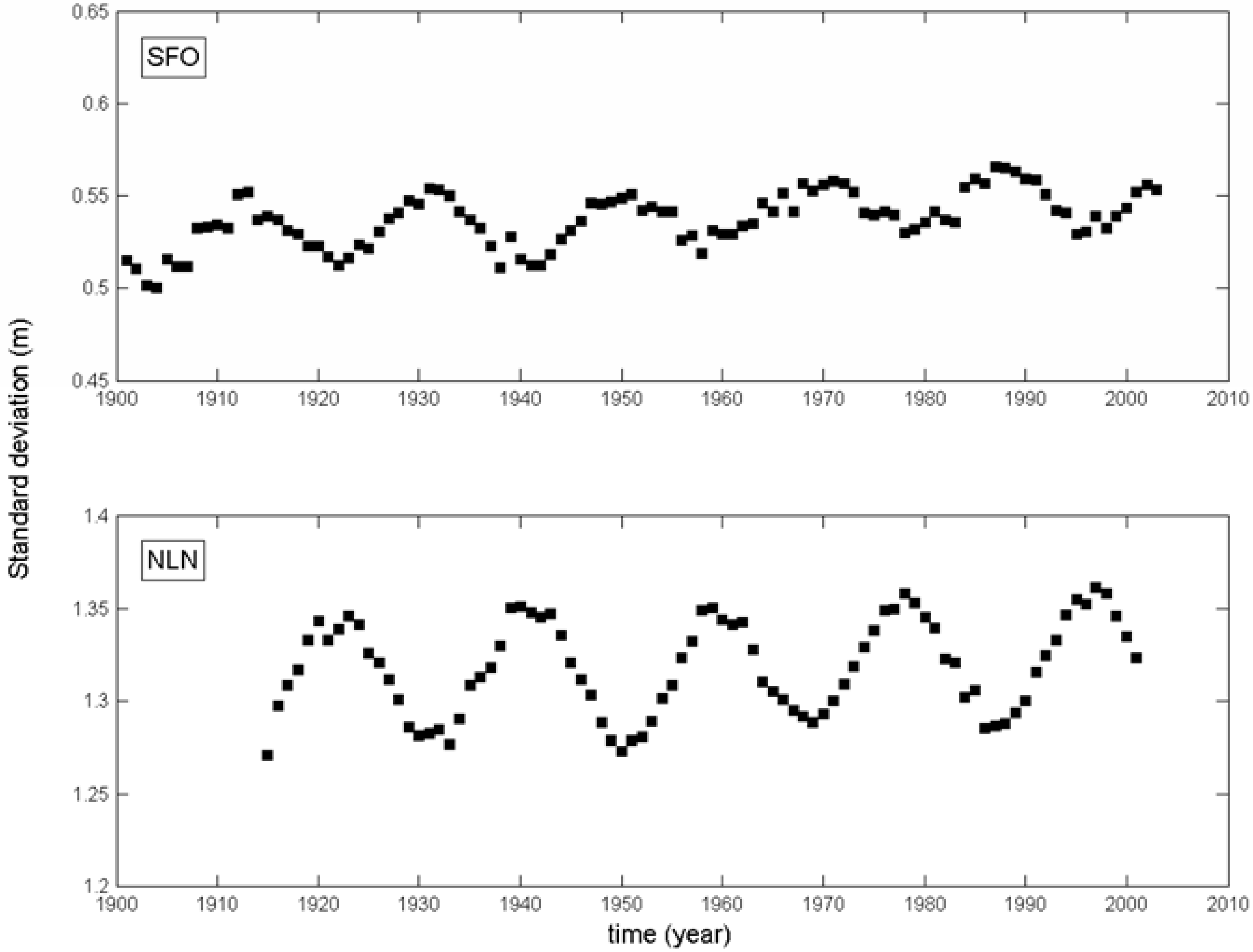
Climatic Variability..



Climatic Variability..



Climatic Variability..



Questions to address:

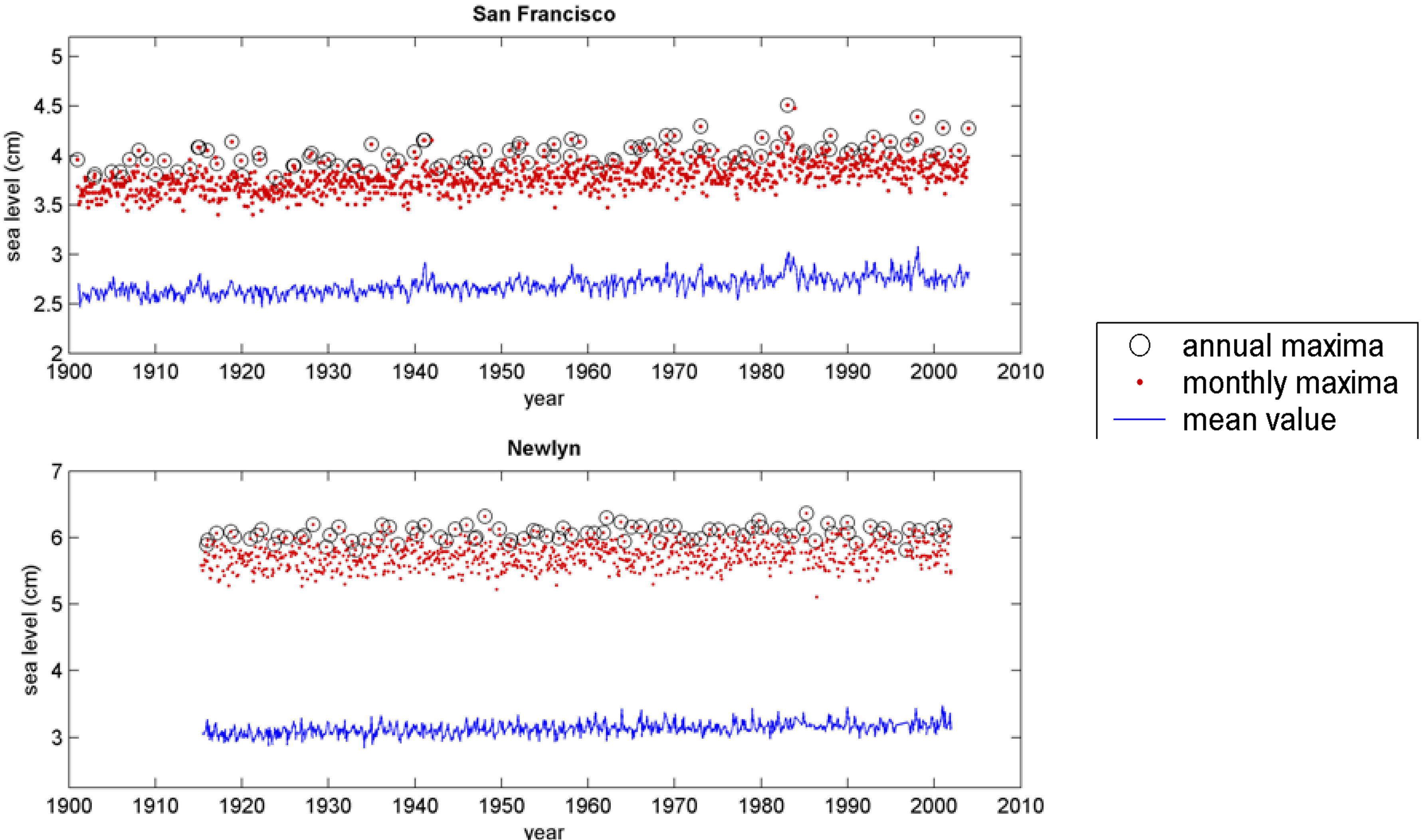
- Are extreme sea levels being affected by climate change ?
- What is the influence of climate-related indices (NAO, SOI,…) on extreme sea levels?
- How is the within a year variation pattern of the extreme sea level events?
- Can we quantify the influence of each physical process in a extreme value model?

OBJECTIVES

• To develop a Statistical Model to characterize rigorously the extreme values of sea level.

• To account for the relevant seasonal-to-interannual variability of extreme sea levels.

❖ Identifying extreme events..



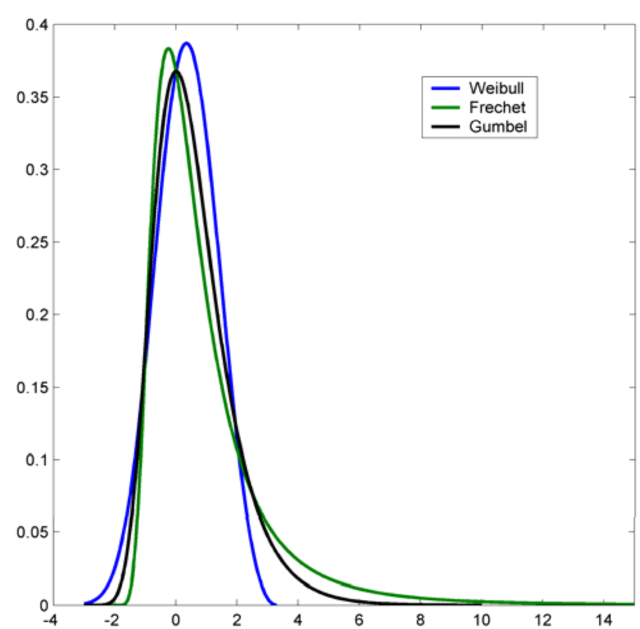
❖ Statistical Model

Extreme Value Theory

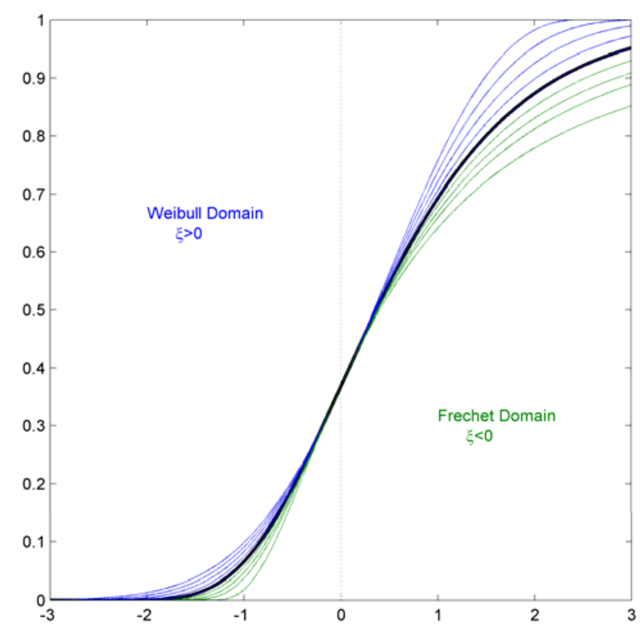
Generalized Extreme Value distribution, GEV

$$F(x;\theta) = \exp\left\{ -\left[1 + \xi \left(\frac{x-\mu}{\psi} \right) \right]^{-1/\xi} \right\}$$

$$\theta = \begin{cases} \mu \rightarrow \textit{location} \\ \psi \rightarrow \textit{scale} \\ \xi \rightarrow \textit{shape} \end{cases}$$



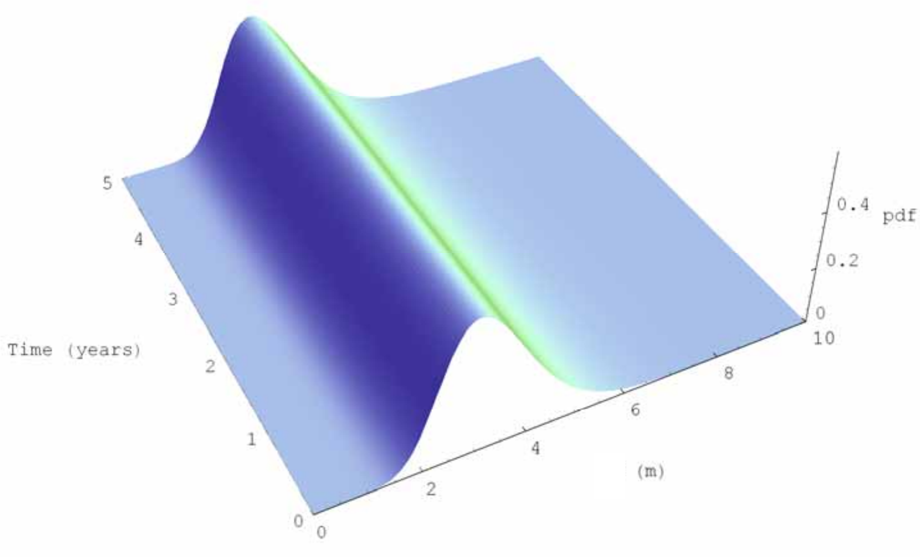
pdf



cdf

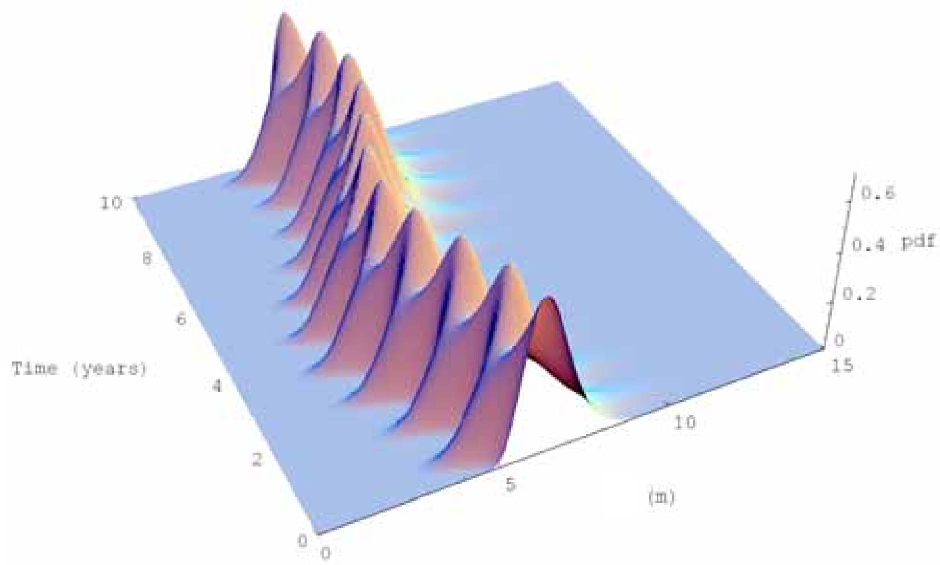
❖ Statistical Model

Stationary process



$$\begin{pmatrix} \mu = cte \\ \psi = cte \\ \xi = cte \end{pmatrix}$$

Non-Stationary process



$$\begin{pmatrix} \mu = \mu_t = f(time) \\ \psi = \psi_t = f(time) \\ \xi = \xi_t = f(time) \end{pmatrix}$$

The probability of an extreme event of a certain magnitude, varies through time

Parameterization

$$\begin{cases} \mu(t) = \mu_{SLR}(t) + \mu_S(t)\exp[\mu_{LT}(t)] + \mu_N(t) + \mu_{CLI}(t) \\ \psi(t) = \psi_S(t)\exp[\psi_{LT}(t)] + \psi_N(t) + \psi_{CLI}(t) \\ \xi(t) = \xi_S(t)\exp[\xi_{LT}(t)] \end{cases}$$

Physical processes	Parameters: Location	Scale	Shape
Seasonality	$\mu_S(t) = \beta_0 + \sum_{i=1}^{P_\mu}[\beta_{2i-1}\cos(2i\pi t) + \beta_{2i}\sin(2i\pi t)]$ $(\beta_0, \beta_1, \beta_2, \beta_3, \beta_4, \dots)$	$\psi_S(t) = \alpha_0 + \sum_{i=1}^{P_\psi}[\alpha_{2i-1}\cos(2i\pi t) + \alpha_{2i}\sin(2i\pi t)]$ $(\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \dots)$	$\xi_S(t) = \gamma_0 + \sum_{i=1}^{P_\xi}[\gamma_{2i-1}\cos(2i\pi t) + \gamma_{2i}\sin(2i\pi t)]$ $(\gamma_0, \gamma_1, \gamma_2, \gamma_3, \gamma_4)$
Sea Level Rise	$\mu_{SLR}(t) = \beta_{SLR} t$ (β_{SLR})	-	-
Nodal Cycle	$\mu_N(t) = \beta_{N_1}\cos(2\pi t / T_N) + \beta_{N_2}\sin(2\pi t / T_N)$ $(\beta_{N_1}, \beta_{N_2})$	$\psi_N(t) = \alpha_{N_1}\cos(2\pi t / T_N) + \alpha_{N_2}\sin(2\pi t / T_N)$ $(\alpha_{N_1}, \alpha_{N_2})$	-
Climatic patterns	$\mu_{CLI}(t) = \begin{cases} \beta_{SOI} SOI(t) \\ \beta_{NAO} NAO(t) \end{cases}$ $(\beta_{SOI}, \beta_{NAO})$	$\psi_{CLI}(t) = \begin{cases} \alpha_{SOI} SOI(t) \\ \alpha_{NAO} NAO(t) \end{cases}$ $(\alpha_{SOI}, \alpha_{NAO})$	-
Secular Trends	$\mu_{LT}(t) = \beta_{LT} t + \beta_{LT_2} t^2$ $(\beta_{LT}, \beta_{LT_2})$	$\psi_{LT}(t) = \alpha_{LT} t$ (α_{LT})	$\xi_{LT}(t) = \gamma_{LT} t$ (γ_{LT})

IH cantabria INSTITUTO DE HIDRÁULICA AMBIENTAL

METHODOLOGY

Parameterization

Codification

Parabolic trend: 1 0

El Niño: 1 0

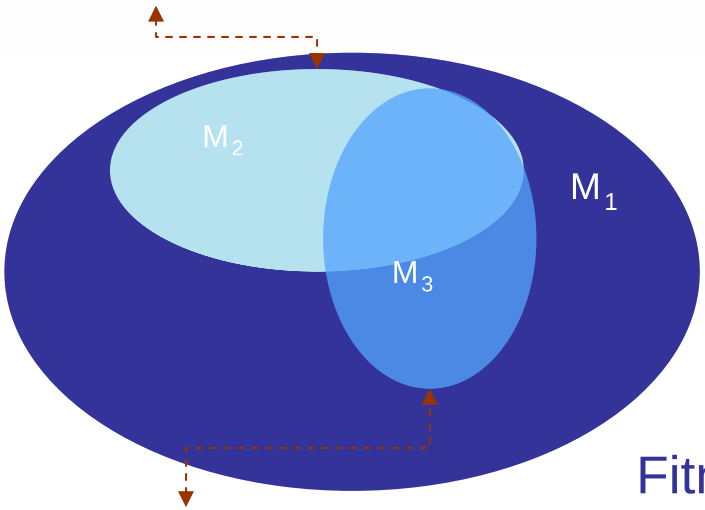
$$\mu(t) = [\beta_0 + \beta_1 \cos(2\pi t) + \beta_2 \sin(2\pi t) + \beta_3 \cos(4\pi t) + \beta_4 \sin(4\pi t)] \exp(\beta_{LT} t + \beta_{LT2} t^2) + \beta_{SOI} SOI(t)$$

Annual cycle: 1 0

Semiannual cycle: 1 0

Linear trend: 1 0

[110010010110000]



[100010010110000]

A possible model: [110011011011111100]

(binary chromosomes)

Automatic Selection
STEPWISE

Fitness: **Maximum Likelihood Estimation**

$$l(\theta \mid t_i, z_i) = -\sum_{i=1}^{m} \left\{ \log \psi(t_i) + (1 + 1/\xi(t_i)) \log\left[1 + \xi(t_i)\left(\frac{z_i - \mu(t_i)}{\psi(t_i)}\right)\right] + \left[1 + \xi(t_i)\left(\frac{z_i - \mu(t_i)}{\psi(t_i)}\right)\right]^{-1/\xi(t_i)} \right\}$$

Statistical Information criterion (BIC & HQ)

Stationary model

Best model

MLE (SE)	SFO-MM0	SFO-MM1	SFO-MM2	SFO-MM3	SFO-MM4	SFO-MM5	SFO-MM7	SFO-MM9	SFO-MM11	SFO-MM12
β_0 (m)	3.728 (0.004)	3.6248 (0.004)	3.626 (0.004)	3.6239 (0.004)	3.6326 (0.003)	3.6353 (0.003)	3.6354 (0.003)	3.6154 (0.005)	3.6146 (0.005)	3.6189 (0.005)
β_1 (m)	—	—	—	—	0.0214 (0.004)	0.0296 (0.004)	0.0269 (0.004)	0.0265 (0.004)	0.0261 (0.004)	0.0271 (0.004)
β_2 (m)	—	—	—	—	0.0302 (0.004)	0.0286 (0.004)	0.0262 (0.004)	0.027 (0.004)	0.0276 (0.004)	0.0283 (0.004)
β_3 (m)	—	—	—	—	0.0823 (0.004)	0.0856 (0.004)	0.0858 (0.004)	0.0842 (0.004)	0.0855 (0.004)	0.0854 (0.003)
β_4 (m)	—	—	—	—	0.0497 (0.004)	0.048 (0.003)	0.048 (0.003)	0.0489 (0.003)	0.0475 (0.003)	0.0488 (0.003)
β_{LT} (yr^{-1})	—	—	—	—	—	—	—	0.000 09 (0.000 02)	0.000 09 (0.000 02)	0.000 09 (0.000 02)
β_{N_1} (m)	—	—	0.0064 (0.005)	—	0.0087 (0.004)	0.0096 (0.003)	0.0095 (0.003)	0.0103 (0.003)	0.0101 (0.003)	0.0071 (0.003)
β_{N_2} (m)	—	—	0.0199 (0.005)	—	0.0201 (0.004)	0.0203 (0.003)	0.021 (0.003)	0.0214 (0.003)	0.0198 (0.003)	0.0236 (0.003)
β_{SOI} (m $unit^{-1}$)	—	—	—	0.0304 (0.004)	—	—	—	—	—	0.0237 (0.003)
α_0 (m)	0.1364 (0.003)	0.1144 (0.003)	0.1133 (0.002)	0.1121 (0.003)	0.0865 (0.002)	0.0858 (0.002)	0.0857 (0.002)	0.085 (0.002)	0.0776 (0.003)	0.0735 (0.003)
α_1 (m)	—	—	—	—	—	0.0274 (0.002)	0.0261 (0.003)	0.0253 (0.002)	0.023 (0.002)	0.0186 (0.002)
α_2 (m)	—	—	—	—	—	0.0072 (0.002)	0.0084 (0.003)	0.008 (0.003)	0.0083 (0.002)	0.0072 (0.003)
α_{LT} (yr^{-1})	—	—	—	—	—	—	—	—	0.0018 (0.000 76)	0.002 (0.000 75)
γ_0	0.1319 (0.013)	0.1112 (0.012)	0.1077 (0.014)	0.1193 (0.012)	0.0743 (0.015)	0.1177 (0.015)	0.1307 (0.016)	0.1339 (0.016)	0.1294 (0.017)	0.1328 (0.017)
γ_1	—	—	—	—	—	—	0.0461 (0.02)	0.0613 (0.02)	0.0686 (0.022)	0.0768 (0.021)
γ_2	—	—	—	—	—	—	0.0519 (0.026)	0.0441 (0.025)	0.0421 (0.026)	0.0325 (0.025)
ℓ	315.71	818.61	828.17	848.4	1138.51	1207.67	1211.63	1224.14	1227.66	1267.21
p	3	3	5	4	9	11	13	14	15	16
AIC	625.42	1631.22	1646.34	1688.80	2259.02	2393.34	2397.26	2420.28	2425.32	2502.42
BIC	610.06	1615.87	1620.74	1668.32	2212.97	2337.05	2330.74	2348.65	2348.55	2420.53

Best model

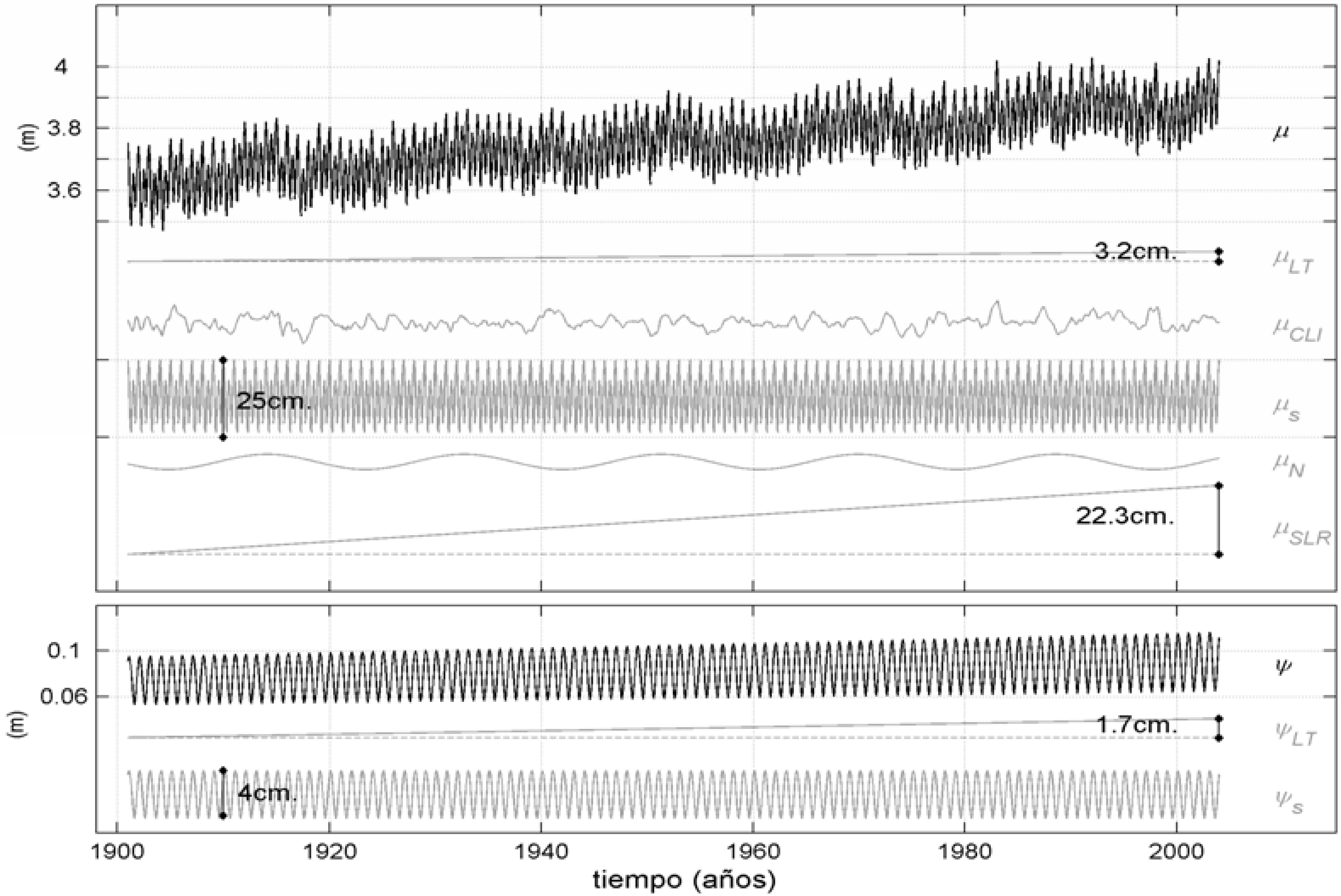
$$\mu(t) = \beta_{SLR}t + [\beta_0 + \beta_1 \cos(2\pi t) + \beta_2 \sin(2\pi t) + \beta_3 \cos(4\pi t) + \beta_4 \sin(4\pi t)]e^{\beta_{LT}t} + \beta_{N_1} \cos(2\pi t/T_N)$$

$$+ \beta_{N_2} \sin(2\pi t/T_N) + \beta_{SOI}\, \mathrm{SOI}(t)$$

$$\psi(t) = [\alpha_0 + \alpha_1 \cos(2\pi t) + \alpha_2 \sin(2\pi t)]e^{\alpha_{LT}t}$$

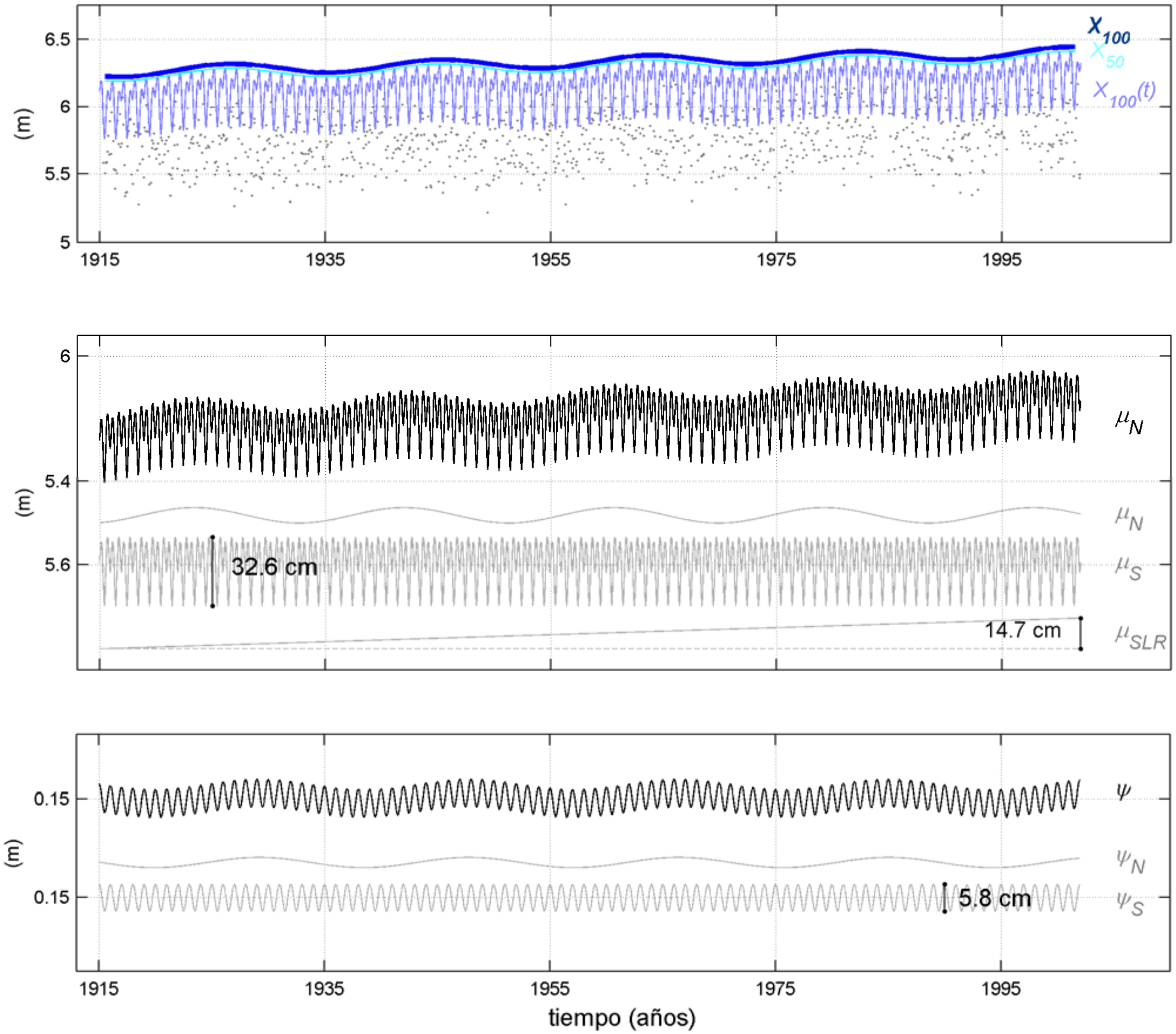
$$\xi(t) = [\gamma_0 + \gamma_1 \cos(2\pi t) + \gamma_2 \sin(2\pi t)].$$

San Francisco

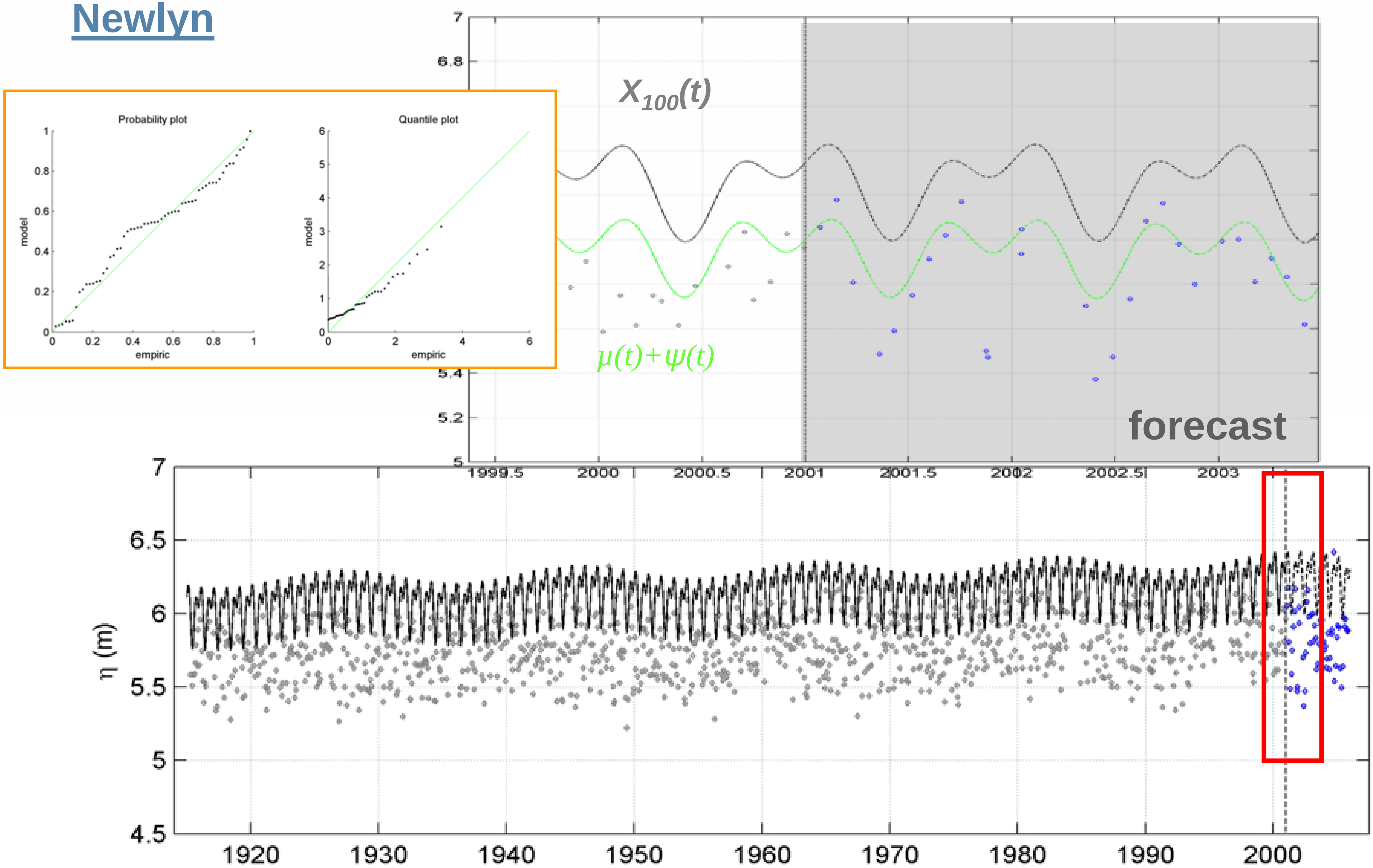


- Increase of 3.2 cm (above the 22.3 cm of SLR)
- Increase of 23% of the variability in the last century!!

Newlyn


6.5
6
5.5
5
(m)
1915
1935
1955
1975
1995
X_{100}
X_{50}
$X_{100}(t)$
6
5.4
5.6
(m)
32.6 cm
14.7 cm
μ_N
μ_N
μ_S
μ_{SLR}
0.15
0.15
(m)
5.8 cm
ψ
ψ_N
ψ_S
1915
1935
1955
1975
1995
tiempo (años)

Newlyn


Probability plot
model
empiric
Quantile plot
$X_{100}(t)$
$\mu(t)+\psi(t)$
forecast
η (m)
1999.5
2000
2000.5
2001
2001.5
2002
2002.5
2003
1920
1930
1940
1950
1960
1970
1980
1990
2000

CONCLUSIONS

- The model is valid for a complete analysis of seasonal-to-interannual sea level extremes providing time dependent quantiles and confidence intervals

- The modelling of the different time scales makes for a better understanding of recent secular trends of extreme climate events, which are one of the main concerns nowadays

- The statistical model is able to predict in the short-term time scale (for example in the next 12 months) the probability of exceedence of a certain extreme sea level value.

References:
Méndez, F.J., Menéndez, M., Luceño, A., Losada, I.J. (2007) Analyzing monthly extreme sea levels with a time-dependent GEV model, Journal of Atmospheric and Oceanic Technology, 24, 5, 894–911.

FORECASTING THE SEASONAL TO INTERANNUAL VARIABILITY OF EXTREME SEA LEVELS

Melisa Menéndez, Fernando Méndez, Iñigo J. Losada

Instituto de Hidráulica Ambiental, IHCantabria,
Universidad de Cantabria